On the Pendulum operations about to be undertaken by the Great Trigonometrical Survey of India; with a sketch of the theory of their application to the determination of the earth's figure, and an account of some of the principal observations hitherto made.—*By* Capt. J. P. Basevi, *R. E., 1st Assistant, Great Trigonometrical Survey of India.*

[Received 29th July, 1865.]

Whilst Lieut.-Colonel Walker, R. E., the Superintendent of the Trigonometrical Survey, was in England last year, General Sabine, the President of the Royal Society, solicited his attention to the importance of making a series of Pendulum observations at the stations of the Great Indian arc, of a similar nature to those made by Captain Kater at the stations of the English arc, and by himself, Captain Henry Foster and others in various parts of both the Northern and Southern hemispheres. Pendulum observations were made on the French arc by Arago, Biot and Mathien early in this century; it is also the intention of the Russian Government to have them made at the principal stations of the Russian arc: moreover there is hardly an instance of the measure of an arc which has not been accompanied by such observations.

General Sabine offered to assist by placing at the disposal of the India Board the pendulums, clocks, and apparatus which he had employed in his own operations; and he added that, should the India Board desire any opinion from the Royal Society on the subject, he would assemble a Committee for the purpose.

Colonel Walker drew up a scheme and estimate of the probable expense, and submitted it with General Sabine's letter for the approval of the Secretary of State for India, who, acting on General Sabine's suggestion, requested the Royal Society to report on the plan of operations proposed by Colonel Walker.

The President accordingly called for opinions from several distinguished Fellows, viz. Professors Challis, W. H. Miller, Stokes, H. J. S. Smith, Dr. Robinson, Sir G. Everest, and Sir John Herschel; all in their replies were agreed on the scientific value of the operations, and all, with the exception of Sir George Everest,* approved of the proposed plan of carrying them out; several made very valuable suggestions.

The Secretary of State in Council consequently sanctioned the experiments, and on Colonel Walker's recommendations he directed Captain Basevi, R. E., who was then in England on furlough, to proceed to Kew to learn the use of the Pendulum and apparatus, with the view of his conducting the experiments in India.

Before detailing the proposed operations, a sketch of the theory, and of what has hitherto been done in the way of Pendulum experiments, may be interesting. The application of Pendulum experiments to determine the figure of the earth, is based upon a theorem demonstrated by Clairaut, which may be stated thus, that the sum of the ellipticity† of the earth, and the fraction expressing the ratio of the increase of gravity to the equatorial gravity is a constant quantity, and is equal to $\frac{5}{2}$ of the ratio of the centrifugal force to the force of gravity at the equator. Hence by ascertaining the difference between the polar and equatorial gravity, or, which is the same thing, the progressive increase in the force of gravity in going from the equator towards the pole, the ellipticity of the earth is at once determined.

It is proved in mechanics that the forces of gravity, at any two stations on the earth's surface, are proportional to the lengths of the seconds Pendulum at those stations, or to the squares of the number of vibrations made by the same pendulum in any given time, one solar day for instance. Here is at once an easy means of determining the variations in the force of gravity, and the solution of the problem of the earth's ellipticity is reduced to the measure of the length of the seconds pendulum at a number of points on the earth's surface, or, as has been most generally done, to the observation of the number of oscillations made by the same pendulum in a mean solar day.

This theory, however, supposes the pendulum to be a "simple pendulum" that is, to consist of a material point suspended by a string without weight, which is, of course a practical impossibility; but as

* Sir G. Everest proposed to employ only the Pendulum of an astronomical clock, but this method is objectionable, as the Pendulum cannot be said to be acted on solely by gravity.

† The ellipticity or compression, as it is sometimes called, is the fraction whose numerator is the difference between the polar and equatorial semi-diameters, and the denominator is the equatorial semi-diameter.

it is always possible to calculate the length of the simple pendulum which would vibrate in the same time as a given compound pendulum, the latter may be used for precisely the same purposes as the former.

Besides this, there are several other conditions supposed to hold good, which in practice are never attained, viz. the arc of vibration has been assumed to be indefinitely small, the length of pendulum to be constant, *i. e.* unaffected by temperature, and the oscillations made in vacuo and at the level of the sea. Corrections have therefore to be computed and applied to the observations, for each of these assumptions.

The time of vibration* in a circular arc is expressed in terms of the length of the pendulum, the force of gravity, and a series of ascending powers of the arc of vibration. The arc is always small, but still not so small that the terms depending on it can be wholly neglected; the first term, however, of the series is all that is ever appreciable in practice. Again, the observations are generally continued for a considerable time, and the change in the arc of vibration has to be taken into account. It has been shewn mathematically, on a certain supposition regarding the resistance of the air, and found to be the case practically, that the arc decreases in a geometric ratio, whilst the times increase in an arithmetic ratio, and on this principle the correction† to the observed time of oscillation is computed.

Secondly, a correction must be applied for the temperature of the pendulum: a change of temperature will, of course, by altering the length of the pendulum, affect the time of its vibration. This cor-

* $$t = \pi\sqrt{\frac{l}{g}}\left\{1 + \left(\frac{1}{2}\right)^2 \text{Sin}^2\frac{a}{2} + \left(\frac{1.3}{2.4}\right)^2\left(\text{Sin}^2\frac{a}{2}\right)^2 + \ldots \ldots \left(\frac{1.3.5.\ldots(2n\text{-}1)}{2.4.6.\ldots 2n}\right)^2\left(\text{Sin}^2\frac{a}{2}\right)^n\right\}$$

in which t = time of one oscillation.
π = semi-circumference of a circle whose radius is unity.
l = length of the Pendulum.
g = force of gravity.
a = arc of semi-vibration.

† The formula for this correction is

$$n.\frac{M.}{32}\ \frac{\text{Sin}(A + a)\ \text{Sin}(A\text{-}a)}{\text{Log Sin A} - \text{Log Sin } a}$$ in which

n = number of oscillations made in a day; M = log *i. e.* modulus = 0·4342945; *A* the *initial* and a the final semi-arcs of vibration. Correction always additive.

rection* must be determined experimentally. Captain Kater immersed his pendulum in fluids of different temperatures, and measured with a micrometric arrangement the alterations in its length. Captain (now General) Sabine observed the change in the number of vibrations made by a pendulum in different temperatures. This is the most direct method of obtaining the correction undoubtedly, but everything depends on the perfect compensation of the clock pendulum with which it is compared.

Thirdly, the formula is only true for observations in a vacuum, and as observations have generally been made in air, or at all events only in a partial vacuum, the effect of the air has to be taken into account. This effect is to diminish the weight of the pendulum by the weight of the air displaced, or to diminish the apparent force of gravity in the same proportion. In the very large majority of observations, the correction has been computed on this consideration solely; but Bessel demonstrated in 1828† that this correction was insufficient, inasmuch as a portion of the surrounding air was set in motion by, and moved with, the pendulum so as to become part of the moving mass. The correction for this can only be determined practically, as by swinging the pendulum in "media" of different densities. It depends chiefly on the form of the pendulum. As this correction "reduction to a vacuum" or "buoyancy correction" as it is

* According to Kater's method—if τ be the standard temperature which is generally taken as 62° Fahrenheit; t the observed temperature of the pendulum: f its factor of expansion for 1° Fahrenheit, then correction $= \frac{1}{2}\ n.\ f.\ (t-\tau)$. positive when $t > \tau$.

† This circumstance was most clearly pointed out by the Chevalier du Buat in 1786, who made a number of experiments with pendulums formed of different substances, but his researches, which created a great sensation at the time, appear to have been completely lost sight of, and to have been unknown even to Borda, who was conducting his experiments, little more than ten years after the publication of Du Buat's results.

The *true* correction for buoyancy Mr. Baily has shown to be (Phil. Trans. 1832)

$C \times \frac{\beta}{1 + .0023\ (t-32°)}$ where β, is the height of Barometer, and t, the temperature during the interval of observation. C is a *constant* for the *same pendulum*, and is determined from the formula

$$C = \frac{N''-N'}{\beta'-\beta''}\ [1 + .0023\ (t°-32°)]$$ in which N' is the number of vibrations in a mean solar day, β' and t' the barometer and thermometer readings, *in air*; and $N,''$ $\beta,''$ t'' the same quantities in a *highly rarified medium*, $t° = \frac{1}{2}\ (t' \times t''.)$

called, depends also on the state of the atmosphere, it is necessary for its calculation, to record the readings of the barometer, when the observations are taken in air.

The last correction is for the height of the station of observation above the mean sea level. The force of gravity varying inversely as the square of the distance from the earth's centre, a pendulum swung at a certain elevation above the sea, will make fewer oscillations in a day than at the level of the sea, and a correction has to be added on this account. Dr. Young, however, demonstrated that the correction computed on this consideration alone, was too large, as it neglected the attraction of the elevated mass itself, and he showed how this might be approximately allowed for.*

The general principle followed in determining the length of the seconds pendulum, is to observe the number of vibrations made by a pendulum of known length, in a mean solar day; then the length of the seconds pendulum is found by multiplying the length of the given pendulum, by the square of the number of its vibrations in a day, and dividing by the square of the number of seconds in a day.

The number of vibrations is generally determined by the method of coincidences. The detached pendulum is placed in front of a good clock, and adjusted to such a length as to gain or lose, (the latter generally) two beats upon the clock in some convenient time, 5 to 10 minutes. Suppose the pendulums to be started togethei, then the longer one of the two will be left behind by the other, the distance between them continually increasing, until at length they will be at opposite extremities of their arcs of vibration at the same moment: the longer pendulum has now lost one oscillation on the shorter one, and both are apparently going at the same rate, but in opposite directions; after a short time they will begin to approach each other, the distance between them gradually diminishing, until they both appear to coincide. It is clear that between two consecutive coincidences the

* This correction is given by the formula $\frac{n}{r} h x$, where n denotes the number of oscillations in a mean solar day, r the radius of the earth at the given station, h the height of the station above the mean level of the sea: x is an unknown quantity determinable from theory; on the assumption that the mean density of the earth is 5.5 and that of the surface 2.5 Dr. Young (Phil. Transactions 1819) showed that the correction for a station on a tract of *table* land would be reduced by $\frac{1}{3}$rd or that the correction $= \frac{2}{3} n h$.

longer pendulum will have lost two oscillations on the shorter one. Hence all that is requisite in practice, is to observe as accurately as possible the intervals between the successive coincidences; the number of vibrations made by the clock pendulum is determined by observations of the sun or stars, and then the number made by the detached pendulum is computed by simple proportion.*

The first pendulum observations of which any account is preserved are those made by Picard at Paris and Uranienburg (Tycho Brahe's observatory) and those by Richer at Cayenne in 1672. These last observations are said to have attracted Newton's attention, as they proved the variation in the length of the seconds pendulum in different latitudes, and it is generally stated that Richer made the discovery by accident. But it appears from Picard's address to the French academy in 1671, that a variation had been already observed, and it is probable that Richer's mission was undertaken partly with a view to throw light on the subject. Picard stated that "from observations made at London, Paris, and Bologna, it would seem as if the seconds pendulum required to be shortened in approaching the equator, but that on the other hand, he is not sufficiently convinced of the accuracy of those measurements, because, at the Hague, the length of the seconds pendulum was found to be quite the same as at Paris, notwithstanding the difference of latitude."†

Near the end of the 18th century, Borda made his celebrated experiments for determining the length of the seconds pendulum at Paris. His apparatus, which is named after him, consisted of a spherical ball of platinum attached by grease to a brass cap which had been truly ground, so as to fit it perfectly. The object of this attachment was to enable the observer to turn the ball round in the cap at pleasure, so as to destroy the effects of unequal density in different parts of it. A fine wire carrying the cap was fastened to the lower end of a small cylinder, passing through the knife edge, which carried on its upper end a small moveable weight, by adjusting which the knife edge and cylinder could be made to vibrate independently in the same

* If $r =$ daily rate of the clock and I the mean interval of the coincidences, then the number of oscillations made by the pendulum in a day $= n$

$$n = \frac{I-2}{I}\,(86400 \pm r)$$ the lower sign is to be used when the clock is *losing*.

† Cosmos Vol. IV. page 25, Sabine's translation.

time as the pendulum, so that their effect might be neglected in computing the length of the simple pendulum. When in use, the knife edge rested upon a steel plate. The number of vibrations per diem, were ascertained by means of a clock, but Borda made a great improvement on the old method of *counting* the coincidences. He fixed a straight edge vertically, so as to coincide with the pendulum wire at rest, when seen through a telescope placed opposite. A cross was made on the bob of the clock pendulum, and the observation consisted in noting the times when the wire and cross disappeared together behind the edge. After a series of coincidences had been observed the length of the pendulum was measured by means of a horizontal steel plate, which was screwed up from below, so as just to touch the ball: then the pendulum was removed, and a bar, whose length had been carefully compared with a standard, inserted in its place. The bar had a T head, of which the lower surface rested on the upper steel plate, and a graduated rod, sliding on the bar, was adjusted to contact with the lower plate. The diameter of the platinum ball was then measured by means of the same slider, by placing it on the steel plate for the purpose; the brass cap and wire were then weighed. The apparatus was enclosed in a glass case, and the temperature was carefully recorded. All necessary corrections were applied, excepting the true one for buoyancy. The whole process, which required very great delicacy, had to be repeated, and the length of the corresponding simple pendulum computed after each series of observations. Borda's pendulum was about 12 feet in length.

His method was followed by M. M. Arago, Biot, and Chaix, at Formentera, the southernmost station of the French arc, with this exception that they used a pendulum of only 3 feet in length. These observations were extended by Biot in 1817 to Leith, and Unst in the Shetlands, and in conjunction with M. Mathien, he observed at Dunkirk, Paris, Clermont, Bordeaux, and Figeac. From these operations, Biot deduced an ellipticity of $\frac{1}{304}$.

In about 1809, Captain Warren made some observations at the Madras observatory with a pendulum formed of a leaden ball suspended by a fibre made from the plantain leaf. The vibrations were counted and an assistant noted the times, from an astronomical clock. In order to measure its length, he attached some glass plates to a wall, and set off on them a scale, transferred from Colonel Lambtons's scale; the length was then measured by a pair of beam compasses. The length of the seconds pendulum was found to be 39.0263 inches of this scale *in air.*

In 1818, Captain Kater published his determination of the length of the seconds pendulum in London at Mr. Browne's house, Portland Place, taken for the purpose of fixing the standard of English measures. His method was founded on the dynamical theorem due to Huyghens, that the centre of oscillation, and axis of suspension, are reciprocal in the same body; that is, if the body be suspended at its centre of oscillation, the former axis of suspension will pass through the new centre of oscillation, and the body will vibrate in the same time as before. The distance from the axis of suspension to the point called centre of oscillation, is equal to the length of the simple pendulum.

In 1822, the English Government sent out an expedition under Captain, now General, Sabine, for the purpose of extending the enquiry commenced by Captain Kater; for both Kater and Biot had come to the conclusion, from a discussion of their experiments, that no decisive result of the earth's ellipticity could be obtained from them, on account of the smallness of the comprised arc, and the variations of local density. Captain Sabine visited thirteen stations between Bahia, S. Lat. 12° 59′ to Spitzbergen N. Lat. 79° 50′. He had with him three pendulums of Kater's invariable pattern, which were all swung at each station. Besides these he had the two clocks and attached pendulums which he had already used on his arctic voyages. His method of observation was similar to Captain Kater's; all the pendulums were swung in London at Mr. Brown's house, both before and after the expedition.

Captain Sabine subsequently determined the difference in the number of vibrations made by an invariable pendulum between London and Paris, London and Greenwich, and London and Attona. He also determined the true buoyancy correction for Kater's convertible pendulum.

In 1825 M. Bessel made his experiments for determining the length of the seconds pendulum at Konigsberg, with an apparatus constructed and partly designed by Repsold the celebrated artist of Hamburg

The apparatus was contrived so as to avoid any uncertainty in the centre of oscillation of the pendulum, as well as any error in the measure of its length, by observing the times of vibration of a pendulum ball suspended alternately by two wires, whose difference in length was known.

A toise was set upright on a narrow horizontal plane firmly fixed to a perpendicular iron bar, and the contrivance by which the pendulums were suspended could be placed either on the horizontal plane, or on the top of the toise itself, so that the effective lengths of the wires differed in the two cases by an amount exactly equal to the length of the toise. The wires, which were of steel, were attached to a thin strip of brass which unwound itself over a small cylinder. The pendulum, thus suspended, described the curve called the evolute of the circle. At the lower end of the iron bar, there was a micrometer screw for measuring small differences in the height of the ball.

The system of observation was as follows. At the commencement of a series of coincidences with the longer pendulum, the thermometers attached to the toise were recorded, and the reading of the lower surface of the ball was taken with the micrometer screw ; the pendulum was then set in motion, and after a sufficient number of coincidences had been observed, the readings of the ball and thermometers were again taken. Exactly the same process was then gone through with the shorter pendulum : then from the times of vibration of the two pendulums, whose absolute lengths were unknown, but whose difference in length was accurately known, the length of the seconds pendulum was easily computed.* There were a great many minute details to be attended to, all of which were carried out with the greatest ingenuity

* Let t_1 & l_1 be times of vibration and length of longer pendulum.

t_2 l_2 " " " shorter "

$l_1 - l_2$ = difference in length $= a$

L = length of seconds' pendulum.

$$\text{Then } t_1^2 = \eta^2 \frac{l_1}{g}, \quad t_2^2 = \eta^2 \frac{l_2}{g}, \quad 1 = \eta^2 \frac{L}{g}$$

$$\therefore \frac{t_1^2}{t_2^2} = \frac{l_1}{l_2}, \quad \frac{t_1^2 - t_2^2}{t_1^2} = \frac{l_1 - l_2}{l_1} \text{ or } \frac{l_1}{t_1^2} = \frac{a}{t_1^2 - t_2^2}$$

$$\text{Again } \frac{1}{t_1^2} = \frac{L}{l_1} \text{ or } L = \frac{l_1}{t_1^2} = \frac{a}{t_1^2 - t_2^2}$$

and nicety, and all conceivable sources of error were considered and their effects computed and allowed for.

The coincidences were observed in a slightly different way from any preceding method. The pendulum was enclosed in a wooden case, faced with glass to keep out currents of air, as well as to preserve as constant a temperature as possible; the clock was placed about 8½ feet in front of the pendulum, and between the two, the object glass of a telescope was adjusted to form an image of the detached pendulum in the plane of the clock pendulum, to enable them both to be seen simultaneously through the observing telescope, which was set up at a distance of about 15 feet. On the wire of the detached pendulum was fixed a small brass cylinder, painted black and called the coincidence cylinder; it weighed something under 4 grains, and could be brought exactly opposite the scale for measuring the arc of vibration.

Captain Kater's pendulum consisted of a bar of plate brass 1.6 inches broad and ⅛th of an inch thick: two knife edges of the hardest steel, attached to solid pieces of brass, were fixed to the bar at a distance of rather more than 39 inches from each other; when the pendulum was in use, these knife edges rested on horizontal planes of agate. At one end of the bar, immediately below the knife edge, was a large flat brass bob firmly soldered to it; and on the bar, between the knife edges, were two sliding weights. The plan of operations was to observe the number of vibrations per diem, made by the pendulum when suspended, first, by one knife edge, and then, by the other; and if these numbers were not identical, to make them so, by means of the sliding weights. The distance between the knife edges, that is, the length of the corresponding simple pendulum, was then measured by a micrometric arrangement. The method of observing the number of vibrations was as follows; to each extremity of the pendulum, a light deal tail-piece, well blackened, was attached; and on the bob of the clock pendulum a white paper disc, equal in diameter to the breadth of the tail-piece, was fastened; the detached pendulum was now placed in front of the clock, and both pendulums being at rest, a telescope was alined, so that the blackened tail-piece exactly covered the paper disc. The telescope was also fitted with a diaphragm, consisting of two perpendicular cheeks, which could be adjusted so as to become tangents to the disc. Now, if both pendulums be set in motion, the detached pendulum vibrating slower than the clock one, the tail-piece will be seen to pass across the diaphragm, followed by the white disc; at each succeeding vibration the disc follows closer and closer, first touching it, and at last becoming completely eclipsed by it. The exact time of this event, called a "disappearance," is noted; after a few more vibrations, the disc will reappear *preceding* the tail-piece; the time of this event, called the "reappearance," is also noted; and the mean of the disappearance and reappearance, is taken as the true time of coincidence. It is immaterial in this method of observation, whether the detached pendulum vibrates faster or slower than the clock pendulum, but it is a *sine quâ non* that its arc of vibration be less. The result, introducing all corrections, except the true one for buoyancy, was 39.13929 inches, which is still the received length, although General Sabine in 1831, showed, by swinging the pendulum in air and in vacuo, that the buoyancy correction was different, according as the heavy weight was above, or below, the plane of suspension.

Captain Kater, in the following year, 1818, made a series of experiments at the principal stations of the English Survey, from Shanklin in the Isle of Wight, to Unst in the Shetlands. He used in these observations a pendulum of a different pattern, known as "Kater's invariable pendulum." With it, it is not possible, nor was it intended, to determine the length of the seconds' pendulum, but it is essentially a differential instrument, and is used for measuring the differences in the number of vibrations at different stations. With these differences, if at any one station the length of the seconds' pendulum has been already determined, the corresponding lengths at the other stations can be ascertained. The invariable pendulum, is of the same dimensions as the convertible one, but is without the second knife edge, and tail-piece, and the sliding weights. The mode of observation is exactly the same. Captain Kater deduced values of the ellipticity, from consecutive pairs of stations; he considered $\frac{1}{304}$ as a probable value (the same as M. Biot's); but he remarks on the difficulty of deriving a satisfactory determination, unless the extreme stations comprise an arc of sufficient extent to render the effects of irregular local attraction insensible.

In 1821-22, some very good observations were made by Mr.

Goldingham, at Madras, and afterwards at a small island called Pulo Gaunsah Lout, lying nearly on the equator in East Longitude 98° 50′. The pendulum used was an invariable one, and observations were first taken with it in London, by Captain Kater. From the observations at Madras and London, Mr. Goldingham deduced an ellipticity of $\frac{1}{297}$.

Captain Basil Hall, assisted by Captain (then Lieutenant) Henry Foster, made a series of experiments with an invariable pendulum in 1820-23, at Galapagos, San Blas (Mexico), Rio Janeiro, and London (Mr. Browne's house). Comparing the results at each of his own stations, with each of Captain Kater's, he deduced ellipticities of $\frac{1}{285}$, $\frac{1}{314}$, and $\frac{1}{302}$.

In 1822, Sir Thomas Brisbane took with him to Paramatta (near Sydney,) an invariable pendulum that had previously been swung in London, at Mr. Browne's house. He deduced ellipticities of $\frac{1}{296}$ and $\frac{1}{304}$, comparing his observations with those of Kater in London and at Unst.

In 1817, the French Government fitted out a scientific expedition under the command of Captain Freycinet, who was furnished with three invariable brass pendulums, one of which was similar to Captain Kater's pattern, and the other two had solid cylindrical rods instead of a flat bar. He had also a fourth pendulum, with a wooden rod formed of two plates of deal firmly clamped together. Instead of a clock he used an astronomical counter, ("compteur astronomique") whose beats could be adjusted to synchronism with those of the pendulum. The counter had a dial, which showed hours, minutes, and seconds, so that by comparing the time shown by this "compteur" with that of a chronometer, he obtained the number of vibrations made by the pendulum in a certain interval, generally an hour or 40 minutes. The pendulums were first swung at Paris, and afterwards at Rio Janeiro, Mauritius, Guam (one of the Ladrone Islands), Mowi (one of the Sandwich Isles), Cape of Good Hope, Port Jackson, Kawak (an island under the line, north of New Guinea) and Malouine or Falkland Isles. Rejecting the determinations at the Mauritius, Guam and Mowi, as they appeared affected to a remarkable degree by local influences, Captain Freycinet deduced an ellipticity of $\frac{1}{289}$ from all four pendulums.

On the return of Captain Freycinet, the French government sent out another expedition under Captain Duperrey. He was supplied with two of Captain Freycinet's brass pendulums, viz. one with a cylindrical rod, and the one on Kater's principle. He observed at six stations, viz. Ascension, Mauritius, Port Jackson, Falkland Isles, Toulon, and Paris. In deducing the ellipticity, he combined his results with those of Freycinet only, and obtained values varying from $\frac{1}{266}$ to $\frac{1}{290}$.

During Ross's voyage to Baffin's Bay in 1818, some observations were taken at Brassa, in the Shetlands, and at Hare Island, with a clock fitted with an invariable pendulum vibrating on a knife edge, which rested on hollow agate cylinders. Observations were repeated at these stations, and a further set taken at Melville Island, on Captain Parry's first voyage to the North Pole in 1819-20. Captain Sabine conducted both these experiments, using the same instruments.

The ellipticity deduced from the experiments at Captain Sabine's stations was $\frac{1}{288 \cdot 4}$, from the same combined with Kater's $\frac{1}{289 \cdot 5}$ and combined again with Biot's $\frac{1}{288 \cdot 6}$ and from a general combination of all of these, $\frac{1}{289 \cdot 1}$. The observations of the *detached* pendulums only were used in these determinations; for though the clock pendulums gave closely coinciding values of ellipticity, still being acted on by other forces than gravity, their results are less reliable, and are only valuable in so far as they afford an independent corroboration of the other results. Captain Sabine was not at first aware of the strict expression for the reduction to a vacuum, but after the publication of Bessel's observations in 1828, he had an apparatus specially constructed, and ascertained the proper correction practically, by swinging his pendulums in air, and in vacuo.

The error from this cause, however, proved to be trifling, owing to his observations being strictly differential, so that only the differences between the corrections by the old and new formulæ entered.

The most widely differing buoyancy corrections at any of his or Captain Kater's stations of observation, computed by the old formula were + 5.75 vibrations at Sierra Leone and + 6.27 vibrations at Spitzbergen, in a mean solar day. These corrections, multiplied by the proper factor, 1.65, to reduce them to the new formula became + 9.52 and + 10.38 vibrations, so that the number of vibrations in a mean solar day at Sierra Leone required to be increased by (9.52 — 5.75)

3.77, and at Spitzbergen by (10.38 — 6.27) 4.11 vibrations. But the acceleration between the stations would only be *increased* by the difference between these numbers, or by 0.44 vibrations. It so happened, however, that even this difference was too large, for in the deduction of the temperature correction, the old buoyancy formula had of course been used; on applying a correction on this account, the above difference required to be *reduced* by 0.36 vibrations so that the whole error on the acceleration of the pendulum between Sierra Leone and Spitzbergen was only + .08 vibrations.

On this scale a black streak was painted, in the middle of which a space was left white, equal to the diameter of the coincidence cylinder, so that when the pendulum was at rest, the cylinder exactly covered it. Again, to the bottom of the clock pendulum a piece of blackened paper was attached, in which a hole had been cut of such a size that when both pendulums were at rest, it exactly coincided with the image of the white space on the black streak: hence when the pendulums were moving in coincidence, the coincidence cylinder was visible through the hole, and completely eclipsed the white space. Bessel's result was expressed in lines of the toise of Peru, the standard used in the measurement of the Peruvian arc.

In publishing these experiments, M. Bessel pointed out the true correction for buoyancy, which he had investigated by swinging in air two spheres of equal diameters, but of different densities, one being of brass and the other of ivory, suspended by a fine steel wire; and again by swinging the same brass sphere first in air and then in water. These experiments showed that the old formula for reducing observations in air to a vacuum gave too small a correction, and that it should be multiplied by a factor.

Mr. Francis Baily made a long series of experiments on the correction for buoyancy, which were published in the Philosophical Transactions for 1832. He used about 80 pendulums, all differing in form, weight, and mode of suspension. From these experiments he deduced factors for pendulums of almost every description that have ever been used, and computed also the weight of the air adhering to each, in other words deduced the *vibrating* specific* gravity of the

* "The vibrating specific gravity of a compound pendulum is *ordinarily* found "as follows; Let d', d'' d''' ... denote the distance of the centre of gravity of each

pendulum. He concluded from all his results, that even if a pendulum is formed of materials having the same specific gravity, yet if it be not of an uniform shape throughout, each distinct portion must be made the subject of a separate computation, in order to determine the correct vibrating specific gravity of the whole body, since each part will be differently affected by the surrounding air.

The last extensive series of experiments were those taken in 1828-31 by Captain Henry Foster, who was sent out on a scientific mission by the Board of Admiralty. He took out with him four invariable pendulums of different metals, two of Captain Kater's pattern, and two of Baily's convertible pattern. These last consisted of a plain straight bar, 2 inches wide, $\frac{1}{2}$ inch thick, and 5 feet $2\frac{1}{2}$ inches long, having two knife edges 39.4 inches apart, but no heavy bob or sliding weights, as in Captain Kater's pattern; the synchronism was adjusted by filing away at one end of the bar; Baily's intention was, that the pendulum should either be used as two different invariable pendulums, or applied as a single convertible one for absolute determinations, at any station. The objection to the form is, that both the knife edges must be exactly perpendicular to the bar, or error is entailed, as the bar is not flexible like Kater's. Captain Foster swung pendulums at all his stations, 14 in number, which were chiefly in the southern hemisphere. He made a set of observations at Mr. Browne's house before the voyage; on the return of the pendulums to England, they were again swung at the same place, but by Mr. Baily, Captain Foster having been most unfortunately drowned in the River Chagres, in February 1831, just as his mission was completed. His observations were reduced by Mr. Baily, who obtained from them an ellipticity of $\frac{1}{289.5}$.

About this time the Russian government sent out an expedition under Captain Lütke, who used an invariable pendulum, formerly used by Captain Basil Hall. He swung it first at Greenwich, and after-

" body respectively from the axis of suspension : w', w'', w''', ... the weight (in air) " of each body : s', s'', s''', ... the specific gravity of each body determined in the " usual manner. Then will the required *vibrating* specific gravity of the pen- " dulum be

$$S = \frac{w'\,d' + w''\,d'' + w'''\,d''' + \ldots}{\frac{w'\,d'}{s'} + \frac{w''\,d''}{s''} + \frac{w'''\,d'''}{s'''} + \ldots}$$

(*Philosophical Transactions*, 1832.)

wards at Ualan, in the Caroline islands, Guam, Bonin island (to the south-east of Japan), at Sitka in Russian North America, at Petropaulowski, Valparaiso, St. Helena, and St. Petersburg. He deduced an ellipticity of $\frac{1}{267}$ from his observations.

Schumacher, the celebrated astronomer of Altona, conducted in 1829-30, a series of experiments with Bessel's apparatus, at the castle of Guldenstein, in order to determine the Danish standard, which was to be a certain fractional part of the length of the seconds pendulum, at the level of the sea, in latitude 45°. In order to estimate the influence of the air, he used, instead of a ball, a hollow cylinder of platinum, made by Repsold, inside which a second solid cylinder, also of platinum, fitted perfectly true. The outer cylinder was closed by covers of the same diameter screwing on to it, which were both perforated; the clamp holding the wire was fastened on to the top, and into the bottom was screwed a point with which the contact was made in measuring the height of the cylinder by the micrometer screw.

The pendulum was swung under four different circumstances, viz. the long pendulum, with and without the inner cylinder, and the short pendulum, also with and without it; and as exactly the same surface was exposed to the air in each case, the influence of it could be computed, which was done by a formula deduced by Bessel. The reduction of the observations was made by Professor Peters. One novelty was introduced, viz. that of computing out the attraction of the ground on which the observations were taken. A square space having a side of 600 toises (1279 yards), in the middle of which the observatory was situated, was subdivided again into 36 squares of 100 toises (213 yards) a side; in each of these borings were made, and specimens of the earth removed and their specific gravities determined; as these were very nearly the same, a mean of the whole was taken. The height of the floor of the pendulum room was 34½ toises (220.6 feet) above the mean sea level, and the attraction of this plateau of the earth's crust introduced a change in the length of the second's pendulum of 0.000215 English inches.

Carlini, whilst measuring the Piedmontese arc in 1821-23, took a series of pendulum experiments at the Hospice on Mount Cenis, with the view of determining the density of the earth. His pendulum was formed of a heavy sphere suspended by a wire, which was attached to a kind of inverted stirrup; in the part corresponding to the foot plate there was fixed a wheel with a sharp edge turning on its axis. This wheel was placed on a grooved plate and formed the knife edge for suspension; the arrangements for observing were similar to Bessel's. Corresponding observations, though not with the same apparatus, were taken by Biot and Mathien at Bordeaux. The result was a density of 4.95.

One more attempt to determine the density of the earth by means of the pendulum was made in 1854 by the Astronomer Royal, Professor Airy, at the Harton Colliery pit. Two invariable pendulums were set up in the same vertical line, one at the top, the other at the bottom of the pit, and their coincidences with the pendulums of two clocks were simultaneously observed, the relative rates of the clocks being determined by a galvanic apparatus. After each series of coincidences the pendulums were interchanged. The distance between the upper and lower pendulums was 1256 feet; a careful description of the intervening strata was prepared and specimens submitted to Professor W. H. Miller who determined their specific gravities. The acceleration of the seconds' pendulum below was 2.24 seconds per diem, and the resulting mean density of the earth was 6.565.

The best value of the earth's ellipticity as yet deduced from pendulum observations is undoubtedly that of Mr. Baily's. He combined all the observations taken with invariable pendulums, and after applying to them all corrections, obtained a mean ellipticity of $\frac{1}{285.3}$. The latest value of the same, from geodetic observations, is Captain Clarke's, R. E. which includes the new Russian arc and is $\frac{1}{294.36}$. The ellipticity obtained from observations of precession and nutation is $\frac{1}{303.3}$ (Airy's tracts).

The apparatus for the Indian experiments, consists of two invariable pendulums on Kater's principle, a vacuum apparatus with air pump for exhausting, an astronomical clock by Shelton, a good battery of thermometers and a transit instrument. Both pendulums have already done good service: one having been used by General Sabine in his extensive range of experiments, the other by Professor Airy in his Harton pit experiments; they cannot be considered, however, to have retained their original length, as their knife edges have been reground.

Each is composed of a bar of plate brass 1.6 inches wide and rather less than an $\frac{1}{8}$th of an inch thick; a strong cross piece of brass is rivetted and soldered to the top to hold the knife edge, which consists of a prism of very hard steel, passing through the bar and adjusted at right angles to its surface. The prism is equilateral in section, but the edge on which it vibrates is ground to an angle of about 120°; the length of the bar from knife edge to the extremity is about 5 feet $1\frac{1}{2}$ inches. At 3′ $2\frac{1}{4}$″ from the knife edge, a flat circular bob, also of brass nicely turned and pierced in the direction of its diameter, is firmly soldered on; the part of the bar beneath the weight, called the tail-piece, which is about 17″ in length, is reduced to a breadth of 0.7 of an inch and is varnished black, in order to contrast better with the white disc on the clock pendulum, in the observation of the coincidences.

The knife edges rest on agate planes set in a solid brass frame, which is provided with three levelling screws. On the outer side of each plane are Y's, which are moveable in a vertical direction by means of an eccentric; the knife edges rest in them when the pendulum is not in use, and by their means the observer is enabled to lower the pendulum down gently so as to bear always on the same parts of the agate planes. Each pendulum has its own set of planes, and will give different results if swung on any others.

It has been decided to swing the Indian pendulums in vacuo, in order to secure the following advantages. When the pendulum has been set in motion, it will vibrate for a whole day; its temperature will be more equable; it will not be disturbed by currents of air; and errors in the formula for the correction for buoyancy are unimportant. The vacuum apparatus consists of a cylinder of sheet copper about 1 foot in diameter and rather more than 5 feet long, with hemispherical caps, the upper one of glass and moveable, the lower one of sheet copper and soldered to the cylinder. The upper end of the cylinder carries a strong brass plate, to which are attached the frames containing the agate planes and a bar of the same metal and shape as the pendulums; placed side by side with a pendulum inside the apparatus, the bar and pendulum will be of the same temperature, and it is evident that thermometers attached to the former will give the required temperature of the latter. Two delicate thermometers are attached to the bar, their bulbs being sunk in the metal at points equidistant from each other and the ends of the bar. The stem of the upper thermometer is inverted, and placed side by side with that of the lower thermometer, in order that they may both be viewed through a moderate sized glass plate let into the cylinder. In the lower part of the cylinder there are four other windows, two on the line of the pendulums, to enable their coincidences to be observed; the other two at right angles to these, to give additional light and enable the observer to ascertain whether the detached pendulum is vibrating truly without wabble. There are two scales fixed at right angles to each other, inside the cylinder, on a level with these windows, one of which is used for measuring the arc of vibration of the pendulum, and the other to measure the distance of the pendulum from the former scale, which is necessary to furnish the correction for parallax in the readings of the arc of vibration: it is useful also in placing the pendulum at a constant distance from the clock, which is found convenient in practice.

The upper 4″ of the cylinder is made of greater thickness than the rest, and at top is a strong projecting flange which is intended to rest on a strong cast iron frame, made in two pieces, so as to grip the cylinder round the thicker part just below the flange; the halves of the frame are then firmly bolted together with nuts and screws. The upper surface of the flange is ground perfectly true to receive a bell glass, the cap already mentioned, which is like the receiver of an ordinary air pump. The eccentric for raising and lowering the pendulum on to the agate planes passes through a stuffing box in the upper part of the cylinder. Motion is imparted to the pendulums by means of a fork and crutch turned by a spindle which passes through another stuffing box.

The clock with which the vibrations are compared is firmly secured to a wall, and the vacuum apparatus is erected in front, at a distance of about 2 feet from it. The diaphragm for limiting the view of the disc is fitted inside the clock case.

The telescope for observing the coincidences is placed on a small masonry pier at a distance of about 8 feet from the vacuum apparatus and is mounted so as to slide laterally on a graduated horizontal bar; it has also a slight vertical motion. The thermometers and barometers are read from alongside of this pillar by means of a cathetometer, viz. a telescope sliding up and down on a vertical rod. The object of this is to obviate the ill effects of any defect in the isolation of the apparatus, as well as the influence of the observer's person on the thermometers.

As the disc on the bob of the clock and the tail-piece of the detached pendulum are too far apart to be viewed simultaneously by the telescope, a lens is placed between them, so as to throw the image of the white disc upon the tail-piece of the pendulum. The vacuum cylinder and all its adjuncts, air pump, &c. were made by Adie, and are the only new portions of the apparatus.

The method of operation is as follows. After setting up the clock, the vacuum apparatus is inserted in the iron frame and suspended either on wooden trestles or masonry piers; the frame is roughly levelled; the temperature bar is fixed in position; the agate planes are screwed on firmly to their bed plate, and are very carefully levelled by means of delicate spirit levels provided for the purpose. A pendulum is now inserted and let down upon its planes, but the clock must not yet be set in motion. The telescope is next set up on the prolongation of the line which passes through the two pendulums, when both are at rest. For this purpose it is moved laterally on its graduated support, until a very small portion of the paper disc, on the bob of the clock pendulum, is visible on one side of the tail-piece of the detached pendulum. The reading is noted, and the telescope is then moved in the opposite direction, until an equal portion of the disc is visible on the other side of the tail-piece; the reading is again noted, and the telescope is set to the mean position. The pendulum is then removed, and the diaphragm in the clock case adjusted, until its checks are tangents to the disc. The pendulum may now be replaced, and nothing remains to be done but to exhaust the air out of the apparatus and to set the pendulums in motion.

The observations are made in exactly the same way as already described in the account of Captain Kater's apparatus; the times of the disappearance and reappearance are both noted, and the mean taken as the true time of coincidence. The arc of vibration is then determined by noting the reading of the arc, when it is cut by the same edge of the tail-piece on each side of the vertical line. The thermometers and barometer are read by means of the cathetometer. It is usual to observe not every coincidence, but the first three consecutive coincidences, and then the 11th, 12th, 13th, then the 21st, 22nd, 23rd, and so on; after observing the first two or three, the times of the after coincidences can be easily computed with sufficient accuracy to intimate when the observer should be ready to note them.

It is intended to have observations made generally along the Great Arc at stations 2″ apart in latitude, and at other points where it may be desirable to obtain data regarding local variations in the intensity of gravity.

The pendulum experiments in this country will afford an independent value of the ellipticity of the Indian arc. It is also hoped that they will throw some light on the existing discordances between the astronomical and geodetic latitudes of the Indian survey.

The amount of the deflections of the plumb line, due to the Himalayas and elevated table lands to the north of India, have been computed by Archdeacon Pratt for the different terminal stations of the Indian arcs; but these determinations are so much in excess of the results of the survey, that it is evident that the effects of the mountain attraction must be in a considerable degree compensated, either by a deficiency of density in the strata to the north, or by an excess of density in the strata to the south of the survey stations.

Now the pendulum can undoubtedly be made the means of showing whether the compensation is to be attributed to either of these causes; for, whilst the effect of a distant range of mountains on the vibrations would be quite inappreciable, any local variation in the density of the underlying strata would show itself most unmistakably; so that by taking observations both at a normal station, and at a few points in its vicinity symmetrically situated around it, should there be any considerable excess or defect in the density of the strata to counteract the disturbance due to the mountain mass, the pendulum observations would not fail to point it out.*

* Professor Stokes remarks in his letter on these operations: "The pendulum no doubt indicates only the vertical component of the disturbing force, "whereas it is the *horizontal component in the plane of the meridian* that affects "the measures of arcs; at any one station, of course, a horizontal disturbance "may exist without a vertical disturbance, and vice versâ; but in a *system* of "stations disturbances of the one kind must necessarily be accompanied by dis-

The Indian operations will eventually be combined with those taken previously with similar instruments in other parts of the world, to deduce the ellipticity of the earth's mean figure. Both Sir John Herschel and Professor Stokes have remarked, in their letters on the proposed Indian operations, that almost all observations hitherto made have been taken at stations either on islands or coasts, so that a series along the centre of a continent is very much needed. A complete set of observations has been already taken at the Kew observatory by Mr. B. Loewy, with the Indian apparatus; and on the completion of the experiments in this country it will be returned to Kew, in order that final observations may be taken, to show whether the pendulums have undergone any change in the interim.

It is to be hoped, however, that so good an opportunity will not be lost of extending these observations to stations easily accessible from India, though not included within its limits. On this head Professor Miller's opinion may be quoted at length, "Much would be added to the value of the observations made at the stations of the Indian survey, if, before the pendulums were brought back to England, observations could be made with them at some other points, especially points nearer to the equator, such, for instance, as the south coast of Ceylon, Singapore, or on the coast of Borneo. Another accessible point, interesting from being in a long line of depression, where a large gravitation might be expected, is Aden."

The intention of the Russian government, to have similar observations made along the Russian arc, has already been alluded to. If, after the return of the pendulums to England, they were to be swung at one of the Russian stations, it will be possible to combine the Russian with the Indian operations, and deduce a value of the earth's ellipticity from exclusively Continental observations, extending from Cape Comorin to the northernmost part of Finmark.

" turbances of the other kind. Indeed it is theoretically possible, from the vertical disturbances, supposed to be known, *actually to calculate* the horizontal " disturbances, and that without assuming anything beyond the law of universal " gravitation. Actually to carry this out, would probably require observations " to be made at stations more numerous than can be thought of, but the fact of " its possiblity shows how severe a check pendulum observations are capable of " exercising on the results of geodetic observations."